$\sqrt{3}\sin(3x) + \cos(3x) = \sqrt{3}$ for $-\pi \le x \le \pi$

we rewrote as $2\sin(3x + \frac{\pi}{6}) = \sqrt{3}$ for $-\frac{17\pi}{6} \le 3x + \frac{\pi}{6} \le \frac{19\pi}{6}$

$\sin(3x + \frac{\pi}{6}) = \frac{\sqrt{3}}{2}$

so $(3x + \frac{\pi}{6}) = 2n\pi + \sin^{-1}(\frac{\sqrt{3}}{2})$ or $(2n+1)\pi - \sin^{-1}(\frac{\sqrt{3}}{2})$, $n \in \mathbb{Z}$

$= 2n\pi + \frac{\pi}{3}$ or $(2n+1)\pi - \frac{\pi}{3}$, $n \in \mathbb{Z}$

Here we can sub in $n=-1, n=0, n=1$ etc to solve for $(3x + \frac{\pi}{6})$ values but let's continue to general solution

$3x + \frac{\pi}{6} = 2n\pi + \frac{\pi}{3}$ or $(2n+1)\pi - \frac{\pi}{3}$, $n \in \mathbb{Z}$

$3x = 2n\pi + \frac{\pi}{3} - \frac{\pi}{6}$ or $(2n+1)\pi - \frac{\pi}{3} - \frac{\pi}{6}$, $n \in \mathbb{Z}$

$3x = 2n\pi + \frac{\pi}{6}$ or $(2n+1)\pi - \frac{\pi}{2}$, $n \in \mathbb{Z}$

$3x = 2n\pi + \frac{\pi}{6}$ or $2n\pi + \pi - \frac{\pi}{2}$, $n \in \mathbb{Z}$

$x = \frac{2n\pi}{3} + \frac{\pi}{18}$ or $\frac{2n\pi}{3} + \frac{\pi}{6}$, $n \in \mathbb{Z}$

$x = \frac{2n\pi}{3} + \frac{\pi}{18}$ or $\frac{2n\pi}{3} + \frac{\pi}{6}$, $n \in \mathbb{Z}$

To find $x \in [-\pi, \pi]$ required by the question,

set $n = 0$; $x = \frac{\pi}{18}$ or $\frac{\pi}{6}$

set $n = 1$; $x = \frac{2\pi}{3} + \frac{\pi}{18}$ or $\frac{2\pi}{3} + \frac{\pi}{6}$

$= \frac{13\pi}{18}$ $= \frac{5\pi}{6}$

set $n = -1$; $x = -\frac{2\pi}{3} + \frac{\pi}{18}$ or $-\frac{2\pi}{3} + \frac{\pi}{6}$

$= \frac{-11\pi}{18}$ $= \frac{-\pi}{2}$